SOFTWARE TOOL ARTICLE

sPop: Age-structured discrete-time population dynamics model in C, Python, and R [version 1; peer review: 2 approved]

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Abstract

This article describes the sPop packages implementing the deterministic and stochastic versions of an age-structured discrete-time population dynamics model. The packages enable mechanistic modelling of a population by monitoring the age and development stage of each individual. Survival and development are included as the main effectors and they progress at a user-defined pace: follow a fixed-rate, delay for a given time, or progress at an age-dependent manner. The model is implemented in C, Python, and R with a uniform design to ease usage and facilitate adoption. Early versions of the model were previously employed for investigating climate-driven population dynamics of the tiger mosquito and the chikungunya disease spread by this vector. The sPop packages presented in this article enable the use of the model in a range of applications extending from vector-borne diseases towards any age-structured population including plant and animal populations, microbial dynamics, host-pathogen interactions, infectious diseases, and other time-delayed epidemiological processes.

Keywords

deterministic, stochastic, vector, population, model, age-specific, survival, development, dynamic, difference equations, C, Python, R

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Introduction: The age-structured population dynamics model

Populations become heterogenous as individuals age and their physical/biochemical characteristics change. This often introduces time delays and has a strong non-linear impact on dynamics¹. As mosquito vectors develop, they react differently to environmental factors². An infection requires a minimum incubation period before it is ready to be transmitted³. Age dependent population stratification should be accounted for in mathematical models to improve population projections and aid in conservation and control.

Incorporating age dependency in mathematical models can be challenging due to the need to keep track of the age of each individual in a population. A common work-around is to introduce predetermined intermittent development stages to account for the different characteristics of each stage and the time it takes to pass from one to another. This approach has been used in various modelling frameworks including deterministic, stochastic, as well as discrete- and continuous-time models⁴⁻⁷. Although intermittent development stages are capable of representing age-structured populations to a certain extent, a large number of age classes are required for accuracy. Consequently, model development becomes a non-trivial task.

Numerous packages including `pobio`⁸, `demogR`⁹, and `bayesPop`¹⁰ have been implemented to facilitate modelling and analysis of age- and stage-structured projection matrix models. As a viable alternative, Kettle and Nutter implemented an R package for age-structured population dynamics, `StagePop`, which offers true time delays in continuous time domain using deterministic delay differential equations¹¹.

Here, I present an alternative age-structured population dynamics model based on the population dynamics and disease-spread models described in Erguler *et al.* 2016¹² and 2017¹³. The model stems from the canonical projection matrix approach and is based on discrete-time difference equations. At present, three implementations of the model exists (the `sPop` packages) for three programming languages, C, Python, and R. The `sPop` packages provide a flexible number of age and development categories, include both deterministic and stochastic dynamics, and offer high-speed simulations to facilitate parameter inference.

The following section describes the theory behind the model and presents the use of each implementation with a commonly encountered case. The same case is modelled with each `sPop` package to emphasise the nuances in their usage. The Use Cases section concludes with a presentation of the `sPop` implementations of a short list of well-known mathematical models from a range of disciplines.

Models and software

The `sPop` packages help to incorporate age-structured species in discrete-time population dynamics and epidemiological models. Survival ($\mathcal{S}$) and development ($\mathcal{D}$) are assumed to act upon each individual in a sequential manner, where development proceeds only if survival is guaranteed for the

duration of an iteration, τ . Survival and development may (i) progress at a fixed propensity, (ii) delay for a given number of iterations, (iii) follow a gamma-distributed (or negative binomial-distributed) life-time, or (iv) halt for a given time period. Here, I adopt the term propensity to refer to either the rate of a deterministic process or the probability of a stochastic one. Although propensity is assumed constant in each iteration, it can be redefined as a user-supplied function, which may depend on age (t_a), number of completed development cycles (t_p), the degree of completion of the present development cycle (t_δ), and any other environmental factors.

Each individual is allowed to stay in the population given that neither death ($\mathcal{X}$ where $\Pr(\mathcal{X}) = 1 - \Pr(\mathcal{S})$) or development occurs during an iteration. In a deterministic setup, a certain fraction of the population will die (r_x) or survive and complete development (r_d) every τ . In a stochastic setup, the number of individuals to die or develop is chosen from a binomial distribution, $\mathcal{B}(n, p)$, where p is the daily probability of death (p_x) or development (p_d) and n is the size of the population. In most experimental setups, development is observed on the condition that individuals survive; therefore, this condition is implicit in the definition of development propensity, *i.e.* $p_d = \Pr(D|S)$ per individual per τ .

At each iteration, t_a and t_δ are incrementally adjusted to keep track of the age and the degree of development, respectively, for each individual. When a batch of individuals complete development, the indicator t_p is adjusted, t_δ is reset to zero, and the batch is removed from the population. When modelling periodic development processes, such as the gonotrophic cycle, the batch can be reintroduced to the population for the subsequent cycle of development.

When propensity is fixed for either $\mathcal{X}$ or $\mathcal{D}$, the time required for the completion of life or development follows a geometric distribution where the size of the population (n) can be described with

$$n_t = n_{t-1} (1 - r_x) (1 - r_d),$$

for the deterministic case, or with

$$\begin{aligned} \Pr(n_t = x) = & \Pr(n_{t-1} = x + 1) p_x (x + 1) \\ & + \Pr(n_{t-1} = x + 1) (1 - p_x) p_d (x + 1) \\ & - \Pr(n_{t-1} = x) (p_x + (1 - p_x) p_d) x, \end{aligned}$$

for the stochastic case, where n_0 is a user-defined initial condition.

When modelling age dependency, the default assumption is that survival is a function of age, t_a , and development is a function of the degree of development, t_δ . The duration of each process can be a fixed number of iterations or can be described by a discrete negative binomial distribution or a continuous gamma distribution. In case of a negative binomial or gamma distribution, the daily propensity of death or development can be calculated as the ratio of the probability that the

process is completed between iteration τ_d and τ_{d+1} to the total probability of surviving or developing for at least τ_d days. For instance, the daily probability of death in a stochastic setup can be written as

$$p_x = \frac{\Pr(\tau_d < x \leq \tau_{d+1})}{\Pr(\tau_d < x)} = \frac{F(\tau_{d+1}; \mu, \sigma) - F(\tau_d; \mu, \sigma)}{1 - F(\tau_d; \mu, \sigma)},$$

where x represents the iteration when death occurs, and $F(\cdot)$ is the cumulative distribution function of either the negative binomial distribution or the gamma distribution. The parameters of each distribution are determined from the desired mean (μ) and standard deviation (σ) of the number of iterations to complete the process. An advantage of the gamma distribution over the negative binomial is its flexibility in accommodating various combinations of mean and standard deviation. However, the negative binomial distribution is restrictive over the minimum allowed standard deviation for a given mean.

Implementation

The R implementation of sPop is available on CRAN as the **albopictus** package (v.0.4) and on the GitHub repository <https://doi.org/10.5281/zenodo.1325095>¹⁴. The C and Python implementations are available as part of the **albopictus** package (v.1.9.3) on PyPI and the GitHub repository <https://doi.org/10.5281/zenodo.1325111>¹⁵. The packages are implemented for R version 3.3.3 and Python version 3.7.0.

This section is reserved for outlining the use of each implementation to model the same theoretical population where both development and survival are age-dependent and gamma-distributed. In addition, the population exhibits a periodic development process with a mean duration of 50 hours and a standard deviation of 10 hours, and survival is a function of the number of development cycles,

$$\begin{aligned} \mu &= \max(240, 480 - 48 t_n) \text{ hours} \\ \sigma &= \mu/10. \end{aligned} \quad (1)$$

R

Before we begin modelling, we load the **albopictus** package in R, and define the survival function as described in Equation 1.

```
R > library(albopictus)
R >
R > death <- function(pop) {
R +   if (nrow(pop)==0) {
R +     return(data.frame(mean=480, sd=48.0))
R +   }
R +   mn <- 480.0 - (48.0 * pop$devcycle)
R +   mn[mn < 240.0] <- 240.0
R +   return(data.frame(mean=mn, sd=0.1*mn))
R + }
```

The function returns a `data.frame` with a desired mean and standard deviation. Next, we initiate a population by calling the initiation routine of the `sPop` class.

```
R > vec <- sPop(stochastic=TRUE, prob="gamma")
```

With this line, we construct a stochastic population model with the gamma distribution as the basis of survival and development. Setting `prob` to `nbinom` selects the negative binomial distribution instead.

In order to introduce the first batch of individuals, we use the `add` method.

```
R > add(vec) <- data.frame(number=1000)
```

By default, age, development cycle, and the duration of development will be set to zero for all individuals. These can be customised by supplying additional fields to the `data.frame`: `age` to set age, `devcycle` to set the number of development cycles, and `development` to set the number of iterations the current development cycle has taken.

We can directly access the population structure of the `sPop` class to inspect the number of individuals grouped with respect to age, development cycle, and the degree of development. Here, we will use these information to calculate the mean and standard deviation of expected lifetime for each age-development group.

```
R > tmp <- death(vec@pop)
```

The following step iterates the population for one time-unit by using the `iterate` method.

```
R > iterate(vec) <- data.frame(dev_mean = 50,
R +                             dev_sd = 10,
R +                             death_mean = tmp$mean,
R +                             death_sd = tmp$sd)
```

By defining `dev_mean` and `dev_sd`, we opt to use the gamma distribution to describe the probability of development. Setting `dev_sd` to zero results in the gamma distribution being discarded and a fixed number of iterations (indicated by `dev_mean`) being assigned for development. Instead, setting `dev` instead of `dev_mean` and `dev_sd` results in a daily constant development probability. Same principles apply for the survival process, where we provide the mean and the standard deviation of the gamma-distribution for each age-development group as calculated by the `death` function. After each iteration, the age and degree of development of the population are updated and the total number of individuals completing development is recorded together with the detailed account of the corresponding age-development groups. We access these data using the `developed` and `devtable` methods, respectively. In addition, the `dead` method returns the number of dead individuals following an iteration.

```
R > d <- developed(vec)
R > add(vec) <- devtable(vec)
```

In this example, we assume a periodic development process; therefore, we introduce all the individuals completing development back to the population using the `add` method.

In addition to `iterate`, the `perturbate` method performs the same procedure to iterate the population for a day, however, leaves age and development unchanged.

This method can be used for modelling external influences such as harvesting and population control, and also emmigration.

Finally, we read the total size of the population using the `size` method.

```
R > s <- size(vec)
```

Python

The Python implementation of the population dynamics model can be imported from the `albopictus` package.

```
Python >>> from albopictus.population import spop
```

We begin by declaring the survival function as in [Equation 1](#).

```
Python >>> def death(pop):
Python ...     if pop.shape[1]==0:
Python ...         return [480.0, 48.0]
Python ...     mn = 480.0 - 48.0 * pop[:,1]
Python ...     mn[mn < 240.0] = 240.0
Python ...     return [mn, 0.1*mn]
```

Unlike the R implementation, the population structure is stored in a `numpy.ndarray` with the following order of columns: age, development cycle, degree of development, and number. Although the initiation step is similar to the R implementation, a two-dimensional list or a `numpy.ndarray` should be supplied to introduce batches of individuals to a population.

```
Python >>> vec = spop(stochastic=True,prob="gamma")
Python >>> vec.add( [ [0, 0, 0, 1000] ] )
```

By using the `add` method above, we introduce 1000 individuals with zero age, development cycle, and degree of development. The population structure is directly accessible, which enables us to calculate a different mean and standard deviation for the gamma-distributed development of each age-development group.

```
Python >>> tmp = death(vec.pop)
```

Next, we iterate the population for one time-unit using the `iterate` method.

```
Python >>> vec.iterate(dev_mean = 50,
Python ...             dev_sd = 10,
Python ...             death_mean = tmp[0],
Python ...             death_sd = tmp[1])
```

In order to read the total number of individuals completing development, the detailed account of the corresponding age-development groups, and the total size of the population, we access the `developed`, `devtable`, and `size` attributes of the `spop` class. Please note that these attributes are overwritten each time the `iterate` method is called. Here, we record the total number of developed individuals and the population size, and reintroduce the developed individuals to the population for the next round of development.

```
Python >>> d = vec.developed
Python >>> vec.add(vec.devtable)
Python >>> s = vec.size
```

The Python implementation of the `iterate` method accepts an additional logical indicator `pause` to prevent updating age and development. If this parameter is supplied and if it is false, the `iterate` method acts as the `perturbate` method of the R implementation.

C

The C implementation of the `sPop` package is further optimised for speed. The source code resides in the `albopictus` package of Python, and it needs to be compiled with the GNU Scientific Library (version 2.1 or later). We begin by locating the package directory and compiling three source files into the object code. Assuming that the file name of our model is `test_spop.c`, we produce the executable with the following.

```
$ gcc -c -o ran_gen.o ${pkgdir}/ran_gen.c
$ gcc -c -o gamma.o ${pkgdir}/gamma.c
$ gcc -c -o spop.o ${pkgdir}/spop.c
$ gcc -I${pkgdir} -lgsl -o test_spop ran_gen.o
gamma.o spop.o test_spop.c
```

where `$pkgdir` is a bash variable holding the package directory. In order to use the package, we need to include the following header files in `test_spop.c`.

```
C #include "ran_gen.h"
C #include "gamma.h"
C #include "spop.h"
```

The first header file defines the routines required for random number generation, and the second one defines the routines for the gamma and negative binomial distributions. The last header file defines the `spop` population structure and the associated functions for initialisation, modification, and garbage collection.

Each age-development group is stored in the `individual_st` data structure,

```
C typedef struct individual_st {
C     unsigned int age;
C     unsigned int devcycle;
C     unsigned int development;
C     sdnum number;
C } individual_data;
```

where the age, development cycle, degree of development, and the number of individuals in each age-development group are stored in `age`, `devcycle`, `development`, and `number` variables in the same order. The `sdnum` is a union data structure holding an unsigned int for a stochastic population or a double for a deterministic population. The `spop` data structure holds an array of individuals (`individuals`), population size (`size`), the number of dead and developed individuals following an iteration (`dead` and `developed`, respectively), a detailed account of developed individuals (`devtable`), an indicator for the probability distribution of

age dependence (`gamma_mode`), a logical indicator for a stochastic or a deterministic model (`stochastic`), and two counters to manage the dynamic size of individuals (`ncat` and `cat`).

```
C typedef struct population_st {
C     individual_data *individuals;
C     sdnum size;
C     sdnum dead;
C     sdnum developed;
C     void *devtable;
C     unsigned char gamma_mode;
C     unsigned char stochastic;
C     unsigned int ncat;
C     unsigned int cat;
C } *spop;
```

Following the procedure in previous sections, we begin implementing the model in `test_spop.c` by declaring the survival function in Equation 1.

```
C void death(const individual_data *ind,
C           double *death_prob,
C           double *death_mean,
C           double *death_sd) {
C     (*death_prob) = 0;
C     (*death_mean) = 480.0 - (ind->devcycle > 4 ?
C     240.0 : 48.0 * ind->devcycle);
C     (*death_sd) = 0.1 * (*death_mean);
C }
```

Please note that the C implementation handles a single age-development group at a time; therefore, the survival function is redesigned accordingly.

Next, we initiate a stochastic model with the gamma distribution as the basis of survival and development using the `spop_init` function with the first parameter set to a logical true.

```
C vec = spop_init(1, MODE_GAMMA_HASH);
```

The macro `MODE_GAMMA_HASH` refers to the optimised implementation of the gamma distribution. Alternatively, `MODE_NBINOM_RAW` and `MODE_GAMMA_RAW` refer to the unoptimised implementations of the binomial and the gamma distributions. Optimisation involves recording previously-used values in a hash table for reuse, however, is memory intensive and should be used with caution. Faster more efficient implementations of the probability distributions are the main concern for future releases.

Having initiated `vec`, we introduce 1000 individuals of zero age with the `spop_add` function.

```
C spop_add(vec, 0, 0, 0, 1000);
```

`spop_add` accepts parameters in the following order:

1. `spop s`: the `spop` data structure
2. unsigned int `age`: the age of individuals
3. unsigned int `devcycle`: the number of development cycles passed
4. unsigned int `development`: the degree of development of individuals

5. `sdnum number`: the number of individuals (unsigned int or double)

In order to iterate the population for one time interval, we use the `spop_iterate` function.

```
C spop_iterate(vec,
C             0,
C             50.0, 10.0,
C             0,
C             0,
C             0, 0,
C             death,
C             0);
```

`spop_iterate` accepts the following parameters in the given order:

1. `spop s`: the `spop` data structure
2. double `dev_prob`: fixed daily development probability (priority over the other development-related parameters)
3. double `dev_mean`: mean development time (gamma or negative binomial)
4. double `dev_sd`: standard deviation of the development time
5. `iter_func dev_fun`: development function (similar to the death function above)
6. double `death_prob`: fixed daily death probability (priority over the other survival-related parameters)
7. double `death_mean`: mean time of death (gamma or negative binomial)
8. double `death_sd`: standard deviation of the time of death
9. `iter_func death_fun`: survival function
10. unsigned char `pause`: logical indicator to prevent updating age and development

Following each iteration, the list of age-development groups that completed their development is stored in the `devtable` variable of the `spop` data structure. In order to reintroduce these individuals back to the population, we use the `spop_popadd` function.

```
C spop_popadd(vec, vec->devtable);
```

It is possible to obtain a summary output of the population structure by using the `spop_print` function, which takes the `spop` data structure as the only parameter. `spop` can be recycled by emptying its contents with the `spop_empty` function.

```
C spop_empty(vec);
```

Finally, in order to clear the memory used by `vec`, we supply its address to the `spop_destroy` function.

```
C spop_destroy(&vec);
```


Model output

All three implementations of the model are given in [Supplementary File 1–Supplementary File 3](#) (`test_spop.R`, `test_spop.py`, and `test_spop.c`). The resulting distribution of the number of individuals completing a development cycle during the first 20 days of simulation is given in [Figure 1](#).

Five cycles of development are clearly seen from the figure, while the population survives for less than 15 days with the survival function defined in [Equation 1](#). Blending of development cycles is apparent and progressive due to the uncertainty in the duration of development (50 hours on average with a standard deviation of 10 hours).

Use cases

This section describes how the Python implementation of `sPop` can be used to model some of the well-known population dynamics models. These models can also be constructed in R and C by following the guideline presented in the previous section.

Nicholson's blowflies

We begin with Nicholson's Blowflies, a classic example of time-delayed stage-structured population model^{11,16}. The model comprises five distinct life-stages and exhibits stable quasi-cyclic oscillations. Although, originally the model was constructed using continuous time-delay equations, we will demonstrate that the `sPop` model adheres well to the observed dynamics and the implementation presented in the `StagePop` package¹¹. Furthermore, we will present a stochastic version of the model, which helps to improve our understanding of the observed variation.

Both the deterministic and stochastic versions of the model are presented in [Supplementary File 4](#) (`case_studies.py`). The

adaptation assumes fixed daily survival and strict development durations, values of which are the same as the original model (Figure 2 in Gurney *et al.* 1983¹⁶). A scaling factor is introduced to calculate hourly instead of daily propensities to improve accuracy on a par with the continuous-time simulations.

As a result, the output of the model is almost identical to the output of the original model (compare [Figure 2\(a\)](#) with the Figure 3a of Gurney *et al.* 1983¹⁶). The six peaks shown between generations 100 and 300 are matched by the stochastic version of the model ([Figure 2\(b\)](#)). However, the amplitude of the oscillations and the relative heights of the minor cycles in each oscillation change drastically at successive generations. Similar fluctuations were also reported by Nicholson (1954)¹⁷ in a laboratory culture of the Australian sheep blowfly (also presented in Gurney *et al.* 1983¹⁶ Figure 1).

Age-structured host-parasite interactions

Another classic example of age dependency in population dynamics was proposed by 18 as a variation of the host-parasite interaction model of 19. The Nicholson-Bailey model considers dynamics in discrete generations where parasites traverse a given area in search of a host. As a result, the number of parasites in the subsequent generation corresponds to the number of hosts parasitised. Hastings introduced age-structure to the host population and assumed that only juvenile hosts are targeted by parasites.

Both the original Nicholson-Bailey model and its age-structured version are implemented in [Supplementary File 4](#) (`case_studies.py`). As shown in [Figure 3\(a\)](#), the original model without age dependency (dashed lines) exhibits oscillations with increasing amplitude around an unstable steady state. The model output matches Figure 10 in Nicholson

Figure 1. Number of individuals completing a development cycle in 20 days. Thick solid line indicates the mean trajectory from the deterministic simulation, while the thin solid line and the dotted lines indicate the median and the 95% range of the stochastic simulation output. The timestep for each iteration is one hour.

(a) Deterministic trajectory

(b) Five stochastic trajectories

Figure 2. Age- and stage-structured model of Nicholson's Blowflies in discrete time. In (a), the deterministic trajectories of eggs (dashed line) and mature adults (solid line) are shown for the corresponding generations. In (b), five stochastic trajectories of mature adults are shown for the same duration.

(1935)¹⁹. Introducing age-structure with a fixed survival rate for host results in the stabilisation of dynamics as reported by Hastings (1984)¹⁸ and seen in [Figure 3\(a\)](#) (solidlines).

Hastings (1984)¹⁸ also discusses the disruptive effect of age-dependent host survival in stability. This is evident in [Figure 3\(b\)](#), where the survival imbalance between young and old individuals drives the dynamics away from the steady state, eventually rendering it unstable. When the age-dependent mortality curve is close to being horizontal, the dynamics closely resembles the Hastings' model with no age limit (solid lines in [Figure 3\(a\)](#) and the dashed line in [Figure 3\(b\)](#)). When mortality is considerably higher in older individuals (red line in [Figure 3\(b\)](#)) or a strict age limit is introduced (green line in [Figure 3\(b\)](#)), the stability is lost and the amplitude of oscillations increase in time.

The Great Plague in Eyam

The final case we study is the severe outbreak of bubonic plague in Eyam (Sheffield, UK) in 1965–1966²⁰. The outbreak was initially modelled by Raggett *et al.* 1982²⁰, and later, a simple deterministic susceptible-infectious-removed (SIR) epidemic model was developed by Brauer *et al.* 2001²¹ to study the outbreak.

An age-structured version of the SIR epidemic model where the infectious stage duration is modelled with a gamma distribution is presented in [Supplementary File 4](#) (case_studies.py). As shown in [Figure 4](#), the number of infectious cases with respect to the number of susceptible individuals, the S-I plane, ([Figure 4\(a\)](#)) and the time trajectory of the outbreak ([Figure 4\(b\)](#)) closely follow the data presented in Brauer *et al.* 2001²¹ Table 9.1.

(a) Age-structured population dynamics

(b) Age-dependent host survival

Figure 3. Age-structured host-parasite interactions of Nicholson-Bailey-Hastings. In (a), the Nicholson-Bailey model (dashed lines) is compared with its age-structured version (solid lines) where parasites choose host in an age-dependent manner. The number of parasites is scaled down to 25% to aid visualisation. In (b), the effect of age-dependent host survival on stability is shown. Mortality rate is given in the inset with respect to age (the number of generations). Dashed line: age-independent mortality; solid green: life expectancy of precisely 10 generations; solid red, teal, and purple: gamma-distributed life expectancy with mean 10 and standard deviation 1.25, 2.5, and 5 generations, respectively.

Figure 5 demonstrates the effect of age-structure on outbreak dynamics by comparing model output with different characteristics of the infectious period. The blue lines indicate the trajectory matching the Great Plague in Eyam. Please note that the corresponding infectious period is only slightly time-dependent where there is a minor difference between the mortalities of newly infected and long-time infectious individuals (plotted in the inset). If the rate of exit from the infectious stage is completely independent from the duration spent in the stage (as is the case with canonical SIR models), the outbreak duration increases (the red lines). In the opposite scenario, where the length of the infectious case is precise, the entire outbreak resolves rapidly as seen in the green trajectory. Since time-dependence has a significant impact on outbreak trajectory, using realistic mathematical formulations to infer outbreak parameters, such as the incubation

period and the rate of infection, becomes a critical step in developing predictive epidemic models.

Summary

The `sPop` packages are designed to facilitate developing time-delayed age-structured population dynamics models in discrete time domain. The underlying population dynamics model incorporates survival, development, and migration, and it can accommodate both deterministic and stochastic dynamics. In order to promote applications, three versions of the model were implemented: the R and Python implementations are aimed at educational and introductory level use, while the C implementation offers further optimisation and high-speed simulations. This paper demonstrated that the model is capable of representing age-structured population dynamics from a

Figure 4. The age-structured SIR model of the Great Plague in Elam. In (a), the simulated numbers of infectious versus susceptible individuals are shown together with the outbreak data. In (b), the number of susceptibles and infectious cases are plotted with respect to time for the duration of the outbreak. Model simulations (solid lines) are compared with the outbreak data (dots).

Figure 5. The effect of age-structure in modelling epidemics. The outbreak trajectory shown in Figure 4(b) (blue lines) is compared with alternative forms of mortality. Red lines indicate time-independence where the gamma distributed infectious period has $\sigma = \mu$. Green lines indicate that the infectious period has a precise length of $\mu = 11.71$ days ($\sigma = 0$).

range of disciplines including insect populations, host-parasite/prey-predator interactions, and infectious disease epidemiology. Future research concerns optimising to limit simulation times and implementing the model in continuous time domain for improved accuracy.

Software and data availability

R implementation of sPop:

- Available from: <https://cran.r-project.org/web/packages/albopictus/>

- Source code: <http://github.com/keruler/albopictusR>
- Archived source code as at time of publication: <https://doi.org/10.5281/zenodo.1325095>¹⁴
- License: GPLv3

C and Python implementation of sPop:

- Available from: <https://pypi.org/project/albopictus/>
- Source code: <https://github.com/keruler/albopictus>
- Archived source code as at time of publication: <https://doi.org/10.5281/zenodo.1325095>¹⁵
- License: GPLv3

All data and source code for running the examples and plotting the figures in this manuscript are provided in the *Supplementary material*.

Competing interests

No competing interests were disclosed.

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Supplementary material

Supplementary File 1: `test_spop.R`. R script file for Section Implementation:R.

[Click here to access the data.](#)

Supplementary File 2: `test_spop.py`. Python script file for Section Implementation:Python.

[Click here to access the data.](#)

Supplementary File 3: `test_spop.c`. C code file for Section Implementation:C.

[Click here to access the data.](#)

Supplementary File 4: `case_studies.py`. Python script file for Section Use Cases.

[Click here to access the data.](#)

Supplementary File 5: `plot_test_spop.R`. R script file for plotting [Figure 1](#).

[Click here to access the data.](#)

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Matthew Silk

Centre for Ecology and Conservation, University of Exeter, Penryn, UK

In this manuscript Erguler introduces the software tool that can be applied to modelling discrete time, age-structured population dynamics in R, Python and C. The basics of using the software in all three programming languages are well covered and the examples of its use and compelling and clearly explained. The code and equations provided seemed accurate and relevant (albeit with the caveat that I am an R user only, and an ecologist rather than mathematical biologist).

I feel that a bit more background provided in the introduction would be beneficial, in particular an overview of the research areas where these models are used (as per the examples) and the reasons that age-structured models can be important to use.

It would also be good to see some of the caveats/constraints of this sort of modelling framework discussed at some point so that a reader was better able to decide if it is a useful tool for them.

Finally, there were a couple of places where I felt a bit more detail would be helpful:

- As someone who doesn't work on insects, a clarification of what constitutes a development cycle would be helpful.
- It would be helpful to more clearly explain the `perturbate()` function and perhaps provide an illustrative example of when it would be used.
- In the Nicholson's blowflies example it is not clear to me from the figure that relative heights of the minor cycles are "changing drastically". I feel this needs to be better depicted or explained.
- When introducing the plague model it would be helpful to explain how it is age-structured and clarify that this applies to the pathogen.

Otherwise I have a set of very minor changes/suggestions.

Introduction:

Paragraph 1 - Suggest editing the first sentence to say that populations are heterogeneous, and one of the ways that they are heterogeneous is because age can cause differences among individuals. This feels like a clearer description to me.

Paragraph 1 - Add "For example," to the start of the second sentence.

Paragraph 1 - Suggest the end of the first paragraph needs some references to support this point.

Paragraph 2 - Sentence starting "This approach" is difficult to read; perhaps split up to cover the deterministic/stochastic and discrete/continuous points separately?

Implementation:

In the R section in paragraph starting "By defining..." - "THE same principles apply...."

In the C section in paragraph starting "The macro..." - "unoptimised implementations of the NEGATIVE binomial"

Use Cases:

Paragraph 1 - Change guideline to guidelines in final sentence.

Age-structured host-parasite interactions - clarifying what Hastings was modelling originally would be helpful - title of reference says predators while in the text it says parasites?

Paragraph starting "Hastings (1984)..." - change resembles to resemble.

Eyam example - It was in 1665-1666 not in 1965-1966 as currently stated in the text.

Summary:

First sentence of the summary reads awkwardly and could do with rephrasing.

Figure 4: One of the points is only partially on each graph - it would be good to change this.

Figure 4 legend: Misspelling of Eyam

Overall this manuscript is clearly written and explained and introduces a potentially useful piece of software available to a wide range of programmers.

Is the rationale for developing the new software tool clearly explained?

Partly

Is the description of the software tool technically sound?

Yes

Are sufficient details of the code, methods and analysis (if applicable) provided to allow replication of the software development and its use by others?

Yes

Is sufficient information provided to allow interpretation of the expected output datasets and any results generated using the tool?

Yes

Are the conclusions about the tool and its performance adequately supported by the findings presented in the article?

Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Disease ecology; Wildlife demography; Animal social structure and behaviour

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Author Response 29 Nov 2018

Kamil Erguler,

I would like to thank Dr Silk for reviewing the manuscript and for his valuable comments and suggestions. I have submitted a revised version with an aim to address the issues raised in his review. My reply to the specific points raised can be seen below. The rest of his suggestions concerning minor corrections have been addressed in the manuscript accordingly.

Addressing specific issues raised in the review:

- I feel that a bit more background provided in the introduction would be beneficial, in particular an overview of the research areas where these models are used (as per the examples) and the reasons that age-structured models can be important to use.
Paragraph 1 - Suggest editing the first sentence to say that populations are heterogeneous, and one of the ways that they are heterogeneous is because age can cause differences among individuals. This feels like a clearer description to me.
Paragraph 1 - Add "For example," to the start of the second sentence.
Paragraph 1 - Suggest the end of the first paragraph needs some references to support this point.

The revised version of the manuscript presents an extended introduction with additional links to the relevant literature and use cases. In addition, the definition of the model is revised and elaborated in the Methods section to convey in a more comprehensive manner the potential areas of use of the proposed modelling strategy. I believe the revised manuscript better describes the approach and places it in the right context.

- It would also be good to see some of the caveats/constraints of this sort of modelling framework discussed at some point so that a reader was better able to decide if it is a useful tool for them.

I added a section named Temporal resolution and accuracy to discuss the effect of time step size on the accuracy of numerical simulations. The section concludes with the assessment that the numerical error increases linearly with increasing step size. I believe that this will help readers to determine an appropriate step size for optimum accuracy given the availability of computational resources.

- As someone who doesn't work on insects, a clarification of what constitutes a development cycle would be helpful.

Development cycle is a concept introduced to the model to accommodate certain biological processes not necessarily related to insect development. In this revision, I used the analogy of human pregnancy to elaborate on the concepts of age (age of the mother), degree of development (duration of her pregnancy), and development cycle (the number of completed development processes which corresponds to the number of births the mother has given in this analogy). I believe that the analogy helps readers better conceptualise these processes and apply them to their work.

- It would be helpful to more clearly explain the perturbate() function and perhaps provide an illustrative example of when it would be used.

In this version of the manuscript, I renamed the method as "perturb", elaborated the discussion on its intended use, and provided a simple example to demonstrate how it can be used to model migration.

- In the Nicholson's blowflies example it is not clear to me from the figure that relative heights of the minor cycles are "changing drastically". I feel this needs to be better depicted or explained.

I thank Dr Silk for pointing out this issue. In this version, I elaborated on the description of the variability in stochastic simulations and its likely correspondence to the observations of laboratory populations. I believe that readers will find this an improvement.

- When introducing the plague model it would be helpful to explain how it is age-structured and clarify that this applies to the pathogen.

In this version of the manuscript, I explained that the canonical SIR model is not age-structured; however, the underlying transmission dynamics might be better represented with an age-structured model. Especially in a case where disease duration or incubation period is of a certain length, the sPop packages are able to track each individual and trigger a state change only for the right ones and only when it is due. I believe that this revised version offers a clearer description of this capacity.

- **Introduction:**

Paragraph 2 - Sentence starting "This approach" is difficult to read; perhaps split up to cover the deterministic/stochastic and discrete/continuous points separately?

Instead of various modelling frameworks, this version of the manuscripts presents examples of various contexts where intermittent stages have been used.

- **Use Cases:**

Age-structured host-parasite interactions - clarifying what Hastings was modelling originally would be helpful - title of reference says predators while in the text it says parasites?

In this version of the manuscript, I stated that Hastings was modelling prey-predator interactions based on the host-parasite model, and presented the analogy between host-parasite and prey-predator interactions as considered by Hastings in his derivations.

- **Summary:**

First sentence of the summary reads awkwardly and could do with rephrasing.

The sentence is rephrased.

- **Figure 4:**

One of the points is only partially on each graph - it would be good to change this.

All Python-generated figures are re-plotted and case_studies.py is updated to improve the visibility of the data points.

Competing Interests: I declare no competing interests.

Reviewer Report 28 August 2018

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Juliane Liepe

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The manuscript "sPop: Age-structured discrete-time population dynamics model in C, Python, and R" by Kamil Erguler provides a comprehensive yet well summarised description of a novel software tool to model age-structured discrete-time population dynamics.

The software tool is implemented in three different programming languages: R, Python and C. All three languages are most commonly used in scientific programming and therefore the presented tools are easily applicable and adaptable for a wide range of users. The manuscript highlights well the differences in implementation for the three programming languages. While R and Python implementations are easy to use and provide an excellent introduction to modellers new to age-structured population dynamics models, the C implementation provides computational speed, which is particularly relevant for parameter optimisation problems in this field.

The manuscript guides the reader through three examples of the Python implementation. For each example, all necessary scripts are provided to reproduce the figures and the results described. While I have not had much experience with age-structured population dynamic models, I found it straight forward to follow the concept of the underlying model described in the manuscript.

Overall this manuscript is well written and provides a hands-on guide to scientists to implement the discussed methods into their existing workflow.

Is the rationale for developing the new software tool clearly explained?

Yes

Is the description of the software tool technically sound?

Yes

Are sufficient details of the code, methods and analysis (if applicable) provided to allow replication of the software development and its use by others?

Yes

Is sufficient information provided to allow interpretation of the expected output datasets and any results generated using the tool?

Yes

Are the conclusions about the tool and its performance adequately supported by the findings presented in the article?

Yes

Competing Interests: No competing interests were disclosed.

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

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