BRIEF REPORT

Verhulst-Pearl growth model versus Malthusian growth model for *in vitro* evaluation of lead removal in wastewater by *Photobacterium sp* [version 1; peer review: peer review discontinued]

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Abstract

Mathematical modeling of microbial populations has a long history of application in the fields of ecology and environmental remediation. In the present study, the Verhulst-Pearl growth model and the Malthusian growth model were used to model and understand the kinetics of *Photobacterium sp* exposed to lead. The results show that goodness of fit of the Verhulst-Pearl growth model was better than the Malthusian growth model. Therefore, the Verhulst-Pearl growth model is considered the best option for providing useful and reliable information about *Photobacterium sp* kinetics growth *in vitro*.

Keywords

Photobacterium sp, Lead, Pb, Verhulst-Pearl growth model, Malthusian growth model

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Introduction

It is well-known that some genera of bacteria, such as *Pseudomonas*, *Xanthomonas*, *Ferroxidans*, *Ralstonia*, *Acidobacillus*¹⁻³, have very interesting capacities as heavy metal remediating agents, since they are capable of extracting metals from solid substrates or transform them into harmless forms. Furthermore, bacteria can be used as bio absorbents for the recovery of metals and for the treatment of industrial effluents^{4,5}. For this reason, the application of these technologies and research for continuous improvements is necessary.

In a previous study, the application of *Photobacterium sp* to remove lead from wastewater was shown to be possible⁶. However, understanding the behavior of this strain within the experimental conditions was not studied. Particularly, computational modeling of the growth kinetics that aims to extract information about interactions between bacteria, nutrients and in this case the pollutant (lead; Pb) could be useful. For this reason, the present study intends to analyze growth of *Photobacterium sp* using two well-known mathematical models, the Verhulst-Pearl growth model and the Malthusian growth model⁷. These models have a long history of application in the fields of ecology, environmental remediation and industrial fermentation⁸.

Methods

The study uses kinetics data collected from the previous study in Quito-Ecuador, which studied the application of *Photobacterium sp* to remove lead from wastewater⁶. Briefly in the previous experiments, *Photobacterium sp* was exposed to two concentrations of Pb (20–100 ppm) in a general growth broth. The experiments were performed in times ranging between 0 and 86 hours, under a controlled temperature (25°C) and unsupervised at room temperature, ranging 5–23°C⁶ at Quito-Ecuador.

Malthusian growth model

This model is often referred to as the mathematical exponential law (MEL). It is extensively adopted in the fields of agronomy, ecology or microbiology. The MEL is therefore seldom referred to as the Malthusian Law, which is a widely accepted view to study Malthusian growth in ecology and microbiology.

The following equation was applied to the kinetics data in the present study:

$$P_t = P_o e^{kt}$$

Where, P(t) is the population in a time, Po is the initial population, k is the specific growth rate and t is the time in hours.

Verhulst-Pearl growth model

For values of t in the domain of real numbers, the S-shaped curve is denoted by the model below. The initial stage of growth is approximately stationary and subsequent is nearly exponential; then, as saturation begins, the growth slows, and at maturity, growth stops as shown:

The following equation was applied to the kinetics data in the present study:

$$P_t = P_o + \frac{A}{1 + e^{-k_m(t-t_0)}}$$

Where, P(t) is the population in a time, Po is the initial population, km is the maximum growth rate and t is the time in hours. t0 represents the time to achieve the middle of the growth and A represents the carrying capacity of the broth tested.

Data analysis

All fittings were performed with sigmaplot 10, using minimum least squares, evaluating the goodness of fit using adjusted R-squared.

Results

Fitting a bacteria growth model aims to understand its kinetics. In Table 1 it is evident that Verhulst-Pearl model has a good performance since a R-squared > 0.90, which is considered as acceptable to model bacteria kinetics curves^{5,9}. In these fitted curves, it is noticeable that room temperature parameters differed drastically from 25°C in carrying capacity (A) and middle time (to) to achieve middle biomass concentration. Therefore, these parameters suggest that 25°C controlled temperature increases the performance in both Pb concentrations. Room temperature seems to reduce the growth considering that Quito-Ecuador is located at 2800 m.a.s.l. Moreover, this model explains the reduction in maximum growth ratio due to the increase of Pb concentration. This effect is present at both temperature conditions (Figure 1).

Table 1. Results of modeling data with Verhulst-Pearl growth model. Temp=temperature, Pb=lead concentration in ppm.

Temp	Lead (ppm)	A	to	km	Po	R-Sq
25°C	20	6.8E+08	33	0.141	6.0E+07	0.987
25°C	100	7.0E+08	41	0.081	8.8E+06	0.975
Room	20	5.2E+08	64	0.113	1.1E+08	0.916
Room	100	5.5E+08	62	0.085	1.4E+08	0.909

Table 2 shows that the Malthusian growth model had an unacceptable goodness of fit, R-squared < 0.90 for all fitted curves. Therefore, this is considered as an unacceptable model for the bacteria kinetics curves tested. In these fitted curves, it is difficult

see differences in temperature and Pb concentration. However, this model provides information about variations in growth ratio (Figure 2). It would be difficult to extend analysis since this model does not fit as well as the Verhulst-Pearl growth model.

Figure 1. Modeling data with Verhulst-Pearl growth model. (A) 25°C, 20ppm lead; (B) Room temperature, 20ppm lead; (C) 25°C, 100ppm lead; and (D) Room temperature, 100ppm lead.

Table 2. Results of modeling data with Malthusian growth model.

Temp	Lead (ppm)	Po	km	R-Sq
25°C	20	2.33E+08	0.016	0.718
25°C	100	1.27E+08	0.022	0.866
Room	20	7.22E+07	0.025	0.886
Room	100	1.05E+08	0.021	0.892

Figure 2. Modeling data with Malthusian growth model. (A) 25°C, 20ppm lead; **(B)** Room temperature, 20ppm lead; **(C)** 25°C, 100ppm lead; and **(D)** Room temperature, 100ppm lead.

Dataset 1. Kinetics data obtained in Ramirez-Cando *et al.*⁶

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Conclusions

Analysing the goodness of fit reveals that the Verhulst-Pearl growth model is the best option to model the kinetics of *Photobacterium* sp instead of the Malthusian growth model, at least in this particular case. *Photobacterium* sp is also suitable to remove lead from water as shown in Ramirez-Cando *et al.*⁶. The Malthusian model has no manner to estimate Carrying capacity since parameters in the model are developed to explain only the exponential growth phase. Moreover, parameters determined by Verhulst-Pearl growth model are very important

in design further research and scaling to preindustrial process in microbiology as well as Monods model¹⁰.

Data availability

Dataset 1: Kinetics data obtained in Ramirez-Cando *et al.*⁶. Uploaded with permission of all the authors¹¹.

Competing interests

No competing interests were disclosed.

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