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BRIEF REPORT

A theoretical model for the prevention of Banana Moko (Musa AAB Simmonds) [version 1; peer review: 2 approved with reservations]

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Abstract

A population simulation model with non-linear ordinary differential equations is presented, which interprets the dynamics of the banana Moko, with prevention of the disease and population of susceptible and infected plants over time. A crop with a variable population of plants and a logistic growth of replanting is assumed, taking into account the maximum capacity of plants in the delimited study area.

Keywords

Mathematical model, Moko, Banana, *Ralstonia solanacearum*, Logistic growth

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Approval Status

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Any reports and responses or comments on the article can be found at the end of the article.

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Introduction

The banana is a fruit of great economic importance and food sovereignty, because it is found in the shopping basket of people across different social strata and because of its nutritional content. However, its production is threatened by re-emerging diseases such as Moko, caused by the bacterium *Ralstonia solanacearum* race 2 phylotype II (Fegan & Prior, 2006), which causes wilting and deterioration of the plant. Symptoms are usually visible when there is a great spread of bacteria and adjacent plants may have already been infected (Viljoen *et al.*, 2016). Moko is a peculiar manifestation of bacterial wilt; it is a quarantine pest that, once inside the host, moves through the vascular bundles. Being a vascular disease, this bacterium not only affects the vegetative part but also the daughter hill, promoting the spread of the disease, which is accelerated by the high optimal minimum, optimal and maximum temperatures of 10°C, 35°C and 41°C, respectively, infecting triploid plantains, heliconia (*Heliconia* spp.) and other ornamental Musacea plants (Jiang *et al.*, 2017).

The development of mathematical models has contributed through the use of a wide range of techniques to the study of epidemics and diseases, helping to answer biological questions and raising new questions related to the epidemiology and ecology of pathogens and the diseases they cause; in most cases, mathematical models lead directly to applications in the control of the disease (Jeger *et al.*, 2018). Some prediction models that calculate the propagation threshold R_0 have evaluated the control of some diseases in plantain and bananas, such as banana wilt by *Xanthomonas* (BXW) using mathematical models, describing a deterministic SI-type epidemic model for control of BXW focusing on the integrated management of the disease through cultural control as in Nannyonga *et al.* (2015), who considered the optimal control strategies associated with the prevention of transmission by the use of contaminated tools. The researchers assumed a model with three modes of transmission: vertical (from the mother plant to its child), horizontal (indirect) from the vector to plant, and through contaminated agricultural tools (Nannyonga *et al.*, 2015).

Likewise, Nakakawa *et al.* (2016) presented a mathematical model for BXW propagated by an insect vector. The mathematical model they formulated takes into account inflorescence infection and vertical transmission from the mother corm to the daughter hills, but not tool-based transmission by humans (Nakakawa *et al.*, 2016). In this context, a dynamic system is formulated based on ordinary two-dimensional differential equations that interprets the dynamics of incidence of banana Moko disease, including prevention and treatment.

The model

A population model with nonlinear ordinary differential equations is presented, which interprets the dynamics of the banana Moko, including a constant rate of disease prevention in the population of susceptible plants over time. A variable population of plants and a logistic growth of replanting are assumed, taking into account the maximum capacity of plants in the study region. The variables and parameters of the model are: $x(t)$, the average number of susceptible banana plants; $y(t)$, the average number of diseased banana plants; and $P(t) = x(t) + y(t)$, total number of banana plants at one time t , shown in Figure 1.

The model parameters are: γ , constant overseeding rate; k , load capacity (maximum capacity) of banana plants in the study region; and β , probability of transmission of infection. Preventive controls are: g , fraction of infected banana plants removed; and f , fraction of susceptible banana plants that receive prevention of contagion of

Figure 1. Banana Moko's disease diagram with prevention.

the bacteria. The dynamic system that interprets the infectious process including prevention and elimination, is formed by the following two nonlinear differential equations:

$$\frac{dx(t)}{dt} = \gamma \left(1 - \frac{x(t) + y(t)}{k} \right) x(t) - \beta \frac{y(t)}{x(t) + y(t)} (1-f)x(t) \equiv h(.) \quad (1)$$

$$\frac{dy(t)}{dt} = \beta \frac{y(t)}{x(t) + y(t)} (1-f)x(t) - gy(t) \equiv \omega(.) \quad (2)$$

With initial conditions $x(0) = x_0$, $y(0) = y_0$, $P(0) = x(0) + y(0)$, $\gamma, k > 0$, $0 < f, g, \beta < 1$, $P \leq k$, $x(t) \equiv x$, and $y(t) \equiv y$.

The region of eco-epidemiological sense is defined where the trajectories of the plant infection dynamics make sense,

$$\Omega = \{(x, y) \in R_+^2 : x + y \leq k\}. \quad (3)$$

Stability and sensitivity analysis

We start by finding the equilibrium populations, the constant solutions of the system, where the population variation of susceptible plants and variation of infected plants become zero, that is, $\frac{dx}{dt} = 0$; $\frac{dy}{dt} = 0$

$$0 = \gamma \left(1 - \frac{x+y}{k} \right) x - \beta \frac{y}{x+y} (1-f)x \quad (4)$$

$$0 = \beta \frac{y}{x+y} (1-f)x - gy \quad (5)$$

We solve this non-linear algebraic system for x and y , determining the equilibrium point with agronomic sense, free of infected plants $E_1 = (k, 0)$. A breakeven point with disease invasion without susceptible plants $E_2 = (0, k)$, and a balance point with susceptible plants and infected plants $E_3 = (\hat{x}, \hat{y})$, with

$$\hat{x} = \frac{k}{\frac{\beta(1-f)}{g}} \left(1 - \frac{\beta \left(\frac{\beta(1-f)}{g} - 1 \right) (1-f)}{\gamma \frac{\beta(1-f)}{g}} \right), \hat{y} = \frac{k \left(\frac{\beta(1-f)}{g} - 1 \right)}{\frac{\beta(1-f)}{g}} \left(1 - \frac{\beta \left(\frac{\beta(1-f)}{g} - 1 \right) (1-f)}{\gamma \frac{\beta(1-f)}{g}} \right) \quad (6)$$

Considering,

$$\xi_0 = \frac{\beta(1-f)}{g} \text{ and } \rho = \frac{\beta \left(\frac{\beta(1-f)}{g} - 1 \right) (1-f)}{\gamma \frac{\beta(1-f)}{g}} = \frac{\beta (\xi_0 - 1) (1-f)}{\gamma \xi_0}, \quad (7)$$

We write x and y like this

$$\hat{x} = \frac{k}{\xi_0} (1-\rho), \quad \hat{y} = \frac{k (\xi_0 - 1)}{\xi_0} (1-\rho)$$

In coexistence of populations $\hat{x} > 0$ and $\hat{y} > 0$, which is true when $\xi_0 > 1$ and $\rho < 1$.

Since $P = x + y$, the total plant population in equilibrium is, $\hat{P} = \hat{x} + \hat{y}$. That is,

$$\hat{P} = \frac{k}{\xi_0} (1-\rho) + \frac{k (\xi_0 - 1)}{\xi_0} (1-\rho) \quad (8)$$

Therefore, $\hat{P} = k(\rho - 1)$.

ξ_0 , indicates the average number of infected plants that an infected plant produces during the infectious period (before being killed) in the population of susceptible plants and is considered the threshold of infected plants. We can consider this threshold as a function that depends on f and g ,

$$\xi_0(f, g) = \frac{\beta(1-f)}{g}. \quad (9)$$

To determine the stability of each equilibrium point (E), we apply the Hartman-Grobman theorem (Perko, 2011), linearizing the system of non-linear Equation (1) – Equation (2), obtaining the linearization matrix (Jacobian matrix) of the form:

$$J(E) = \begin{pmatrix} h_x(E) & h_y(E) \\ \omega_x(E) & \omega_y(E) \end{pmatrix}$$

With the following partial derivative elements,

$$a_{11} = h_x(E) = \gamma - \frac{\gamma}{k}(\hat{x} + \hat{y}) - \frac{\gamma}{k}\hat{x} - \frac{\beta(1-f)\hat{y}^2}{(\hat{x} + \hat{y})^2}$$

$$a_{12} = h_y(E) = -\frac{\gamma}{k}\hat{x} - \frac{\beta(1-f)\hat{x}^2}{(\hat{x} + \hat{y})^2}$$

$$a_{21} = \omega_x(E) = \frac{\beta(1-f)\hat{y}^2}{(\hat{x} + \hat{y})^2}$$

$$a_{22} = \omega_y(E) = \frac{\beta(1-f)\hat{x}^2}{(\hat{x} + \hat{y})^2} - g$$

These elements of the matrix $J(E)$ are the coefficients of the linear system

$$\frac{d}{dt} U = J(E)U \quad (10)$$

Where, $U = (u, v)^t$ (transposed vector).

We analyze the balance points with an agronomic sense $E_1 = (k, 0)$ y $E_3 = (\hat{x}, \hat{y})$. For E_1 , we obtain the Jacobian matrix,

$$J(E_1) = \begin{pmatrix} \lambda & -\{\gamma + \beta(1-f)\} \\ 0 & g(\xi_0 - 1) \end{pmatrix} \quad (11)$$

Because it is a triangular matrix, the eigenvalues ($\lambda_i, i = 1, 2$)

$$\lambda_1 = -\gamma, \quad \lambda_2 = g(\xi_0 - 1)$$

where $\lambda_2 < 0$ since the threshold $\xi_0 < 1$.

We conclude that the free equilibrium point of Moko disease is locally and asymptotically stable.

For case $E_3 = (\hat{x}, \hat{y})$, in matrix (10) we obtain the trace and the determinant of $J(E_3)$, respectively,

$$Tr.J(E_3) = a_{11} + a_{22}; det.J(E_3) = a_{11}a_{22} - a_{12}a_{21}$$

We conclude that the equilibrium point with susceptible plants and infected plants is locally and asymptotically stable if the threshold inequalities (7) and the inequalities are met,

$$\gamma + \frac{\beta(1-f)\hat{x}^2}{(\hat{x} + \hat{y})^2} < \frac{\gamma}{k}(\hat{x} + \hat{y}) + \frac{\gamma}{k}\hat{x} + \frac{\beta(1-f)\hat{y}^2}{(\hat{x} + \hat{y})^2} + g$$

$$\left\{ \frac{\gamma}{k}\hat{x} + \frac{\beta(1-f)\hat{x}^2}{(\hat{x} + \hat{y})^2} \right\} \frac{\beta(1-f)\hat{y}^2}{(\hat{x} + \hat{y})^2} > \left\{ \gamma - \frac{\gamma}{k}(\hat{x} + \hat{y}) - \frac{\gamma}{k}\hat{x} - \frac{\beta(1-f)\hat{y}^2}{(\hat{x} + \hat{y})^2} \right\} \left[g - \frac{\beta(1-f)\hat{x}^2}{(\hat{x} + \hat{y})^2} \right]$$

These analytical results are shown in the phase planes of **Figure 2**, made with Maple 18 software ([free trial available](#); SageMath is an openly available alternative), for different scenarios varying initial conditions.

Results and conclusions

Local sensitivity is a measure of the relative change in a variable when its parameters change ([Chitnis et al., 2008](#); [Hamby, 1994](#); [Rodrigues et al., 2013](#)). That is,

$$I_{\xi_0}^p = \frac{\partial \xi_0}{\partial p} \frac{p}{\xi_0}$$

Where,

$$\xi_0 = \frac{\beta(1-f)}{g} \quad (12)$$

where β, g, f , are previously defined parameters.

The indices of local sensitivity of the epidemic threshold with respect to each parameter are $I_{\xi_0}^{\beta} = 1$, $I_{\xi_0}^f = -0.43$ y $I_{\xi_0}^g = -1$. These values indicate that the parameter that most influences the threshold value is β proportionally and g inversely proportional.

It is concluded that mathematical simulation models are a useful tool for research in banana Moko disease. With them it was determined that the elimination of banana plants infected with the disease plays an essential role in the good agronomic management of the crop.

As a research perspective, consider the problem of a simulation model including a piecewise function for the rate of elimination of infected plants, in the form

$$g = \begin{cases} 0 & \text{si } 0 \leq t < T \\ g_0 & \text{si } t \geq T \end{cases} \quad (13)$$

Where T is the time it takes for agricultural institutions to confirm the presence of plantain Moko and suggest the elimination of infected plants.

Figure 2. Local stability of the susceptible plant population and infected plant population corresponding to $\xi_0 = 7$ y $\xi_0 = 0.79$.

The analysis of the following optimal control problem is also proposed as a research perspective, applying the principle of the Pontryagin maximum (Kopp, 1962).

The functional objective of direct and indirect costs is proposed:

$$J(x, u) = \int_0^T L(x, u) dt = \int_0^T \left[n_1 p_i(t) + \frac{n_2}{2} u_1^2(t) + \frac{n_3}{2} u_2^2(t) \right] dt \quad (14)$$

Linked to the system of differential equations:

$$\frac{dp_s(t)}{dt} = \gamma \left(1 - \frac{p_s(t) + p_i(t)}{k} \right) p_s(t) - \beta \frac{p_i(t)}{p_s(t) + p_i(t)} (1 - u_1(t)) p_s(t) \equiv f_1 \quad (15)$$

$$\frac{dp_i(t)}{dt} = \beta \frac{p_i(t)}{p_s(t) + p_i(t)} (1 - u_1(t)) p_s(t) - u_2(t) p_i(t) \equiv f_2 \quad (16)$$

With initial conditions $p_s(0) = p_{s0}$, $p_i(0) = p_{i0}$, $P(0) = p_{s0} + p_{i0}$, $\gamma, k > 0$, $0 \leq u_1, u_2$, $\beta \leq 1$, $P \leq k$, $n_i > 0$, $i=1,2,3$, $P_s \equiv x$ and $P_i \equiv y$.

It is about finding optimal control $(\bar{u}_1(t), \bar{u}_2(t))$ such that:

$$J(\bar{u}_1(t), \bar{u}_2(t)) = \min_{\Gamma} J(\bar{u}_1(t), \bar{u}_2(t)) \quad (17)$$

Where,

$$\Gamma = \{(\bar{u}_1(t), \bar{u}_2(t)) \in L^2(0, T) : 0 \leq u_1(t) \leq 1, 0 \leq u_2(t) \leq 1\} \quad (18)$$

is the space of admissible controls and L^2 is the space of integrable functions.

Data availability

All data underlying the results are available as part of the article and no additional source data are required.

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Version 1

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Ana Maria Pulecio Montoya

Department of Mathematics and Statistics, University of Nariño, Pasto, Colombia

The manuscript deals with analytical and numerical study of a model of Ordinary Differential Equations to treat banana Moko dynamics. I submit the following suggestions in order to improve this article:

- The abstract needs to explain the main results of the article.
- The authors should explain better the description in the differential equations of the model.
- The authors should explain better how they obtained the equilibrium points and why the equilibrium point E2 is not of interest in the analysis presented.
- Sensitivity analysis should be written as a section and the authors should explain better the results.
- The conclusions should highlight the main results of the model analysis without giving new results or repeating what has already been said.

Is the work clearly and accurately presented and does it cite the current literature?

Partly

Is the study design appropriate and is the work technically sound?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?

Partly

If applicable, is the statistical analysis and its interpretation appropriate?

Not applicable

Are all the source data underlying the results available to ensure full reproducibility?

Yes

Are the conclusions drawn adequately supported by the results?

Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Mathematical biology, applied Mathematics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 23 December 2020

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Dalia M. Muñoz

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The authors present a simulation model to treat banana Moko dynamics, which constitutes an essential topic in food safety. The model seems to be correctly implemented. However, the manuscript lacks some clarity, and more discussion on the results is strongly encouraged. The manuscript needs corrections before any decision. A point by point list with the concerns raised after my revision is listed in the next lines.

1. The abstract just mentioned how they performed the model and some other conditions like replanting. The abstract needs to be strengthened to discuss the main results and the implications of treating banana plants with some prevention measures.
2. The introduction needs to be strengthened to depict the state of the art on the banana Moko study. There exist other simulation models based on differential equations to treat the banana Moko? Why did you choose such a model?

Addressing these aspects contributes to the authors pointing out the scarce literature in this regard and their study's relevance.

3. The following paragraph lacks clarity. Please rewrite it.

Some prediction models that calculate the propagation threshold R_0 have evaluated the control of some diseases in plantain and bananas, such as banana wilt by *Xanthomonas* (BXW) using mathematical models, describing a deterministic SI-type epidemic model for control of BXW focusing on the integrated management of the disease through cultural control as in Nannyonga et al. (2015), who considered the optimal control strategies

associated with the prevention of transmission by the use of contaminated tools. The researchers assumed a model with three modes of transmission: vertical (from the mother plant to its child), horizontal (indirect) from the vector to plant, and through contaminated agricultural tools (Nannyonga et al., 2015).

4. The following sentences should be included in the section Model

Local sensitivity is a measure of the relative change in a variable when its parameters change (Chitnis et al., 2008; Hamby, 1994; Rodrigues et al., 2013). That is....

5. What do authors mean when they talk about agronomic sense?

6. The results need to be expanded, including a better description. In the case of figure two, the authors do not present any description.

7. Conclusions generally present the main findings. Please rearrange it to meet the high-quality standards of the journal.

8. The presentation of the research perspective seems messy and needs to be improved. The authors present a research perspective, but it is not considered in the paper's main body. The author should consider it as part of the analysis. The optimal control problem should be in the Section Model, describe the development in the results and the implications or expected results with this analysis in conclusion.

9. The list of references requires an update to include recent studies.

10. Authors are encouraged to look for other models about diseases in banana crops and discuss their results to see advantages and differences.

Overall, the paper looks interesting, but it needs a major revision before any consideration.

Is the work clearly and accurately presented and does it cite the current literature?

Partly

Is the study design appropriate and is the work technically sound?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?

Yes

If applicable, is the statistical analysis and its interpretation appropriate?

Not applicable

Are all the source data underlying the results available to ensure full reproducibility?

Yes

Are the conclusions drawn adequately supported by the results?

Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Public health, environment, sustainability, environmental management, econometrics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

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